

Natural Convection with Heat Generation in Vertical Porous Annulus with Axial and Radial Magnetic Field

Vivek Shrivastava, Pavan Badami, Mukesh Patil, K.N.Seetharamu

Abstract—Effect of radial and axial magnetic field on natural convection in vertical porous annulus with internal heat generation has been investigated. Thermal equilibrium model has been considered for study. Study has been carried out for inner and outer wall subjected to same constant temperature. Top and bottom wall are insulated and impermeable. Strength of uniform horizontal and vertical field has been varied by varying Hartmann number. Effect of Hartmann number, magnetic field direction, Rayleigh number, Aspect ratio, Radius ratio has been studied. Variations of Nusselt number, streamlines, isotherms have been plotted for all parameters. Finite Element Method has been used to solve governing Partial differential equations.

Index Terms—Magneto hydrodynamics, Natural Convection, Porous annulus, Heat generation, Finite Element Method, Darcy model, MATLAB.

1 INTRODUCTION

Natural convection in porous media has varied application and hence has been studied for a long time. Applications include geothermal energy systems, nuclear reactors, packed bed reactors, electronic cooling, catalytic reactors, food processing etc. MHD or Magneto hydrodynamics deals with study of dynamics of electrically conducting fluid such as plasma, electrolytes, liquid metal. Study of MHD is important as it has varied applications such as MHD generators, geothermal energy extraction, nuclear reactors, MHD pumps etc. Due to varied application, study of MHD in porous annulus becomes imperative.

There is abundant literature available on natural convection in porous media in presence of magnetic field. Barletta, Lazzari, Magyari, Pop [1] have studied mixed convection in porous annulus surrounding and electric cable which give rise to radially varying magnetic field. They have also taken heat generation due to Joule heating and viscous dissipation into account. Venkatachallappa, Do, Sankar [2] have studied effect of magnetic field on double diffusive convection in a porous annulus with constant temperature and concentration boundary condition. Mansour, Chamkha, Mohamed, Abd-El-Aziz, Ahmed [3] have studied unsteady MHD natural convection in an inclined cavity.

They have also studied effect of variation of magnetic field on free convection in square cavity. Hussein, Ahmed, Saha, Hasanpour, Mohammed, Kolsi, Adegun[4] have studied MHD natural convection in trapezoidal enclosure subjected to uniform flux and internal generation or absorption. Sankar, Park, Lopez, Do [5] have worked on effect of discrete heating through Brinkman extended Darcy equation on natural convection in vertical annular cylinder.

They have used implicit finite difference scheme. They have carried out analyses for wide range of Rayleigh and Darcy number and various heater lengths and locations. Sankar and Do [6] have worked on natural convection in vertical annular cylinder considering two flush mounted heat sources while outer wall has been cooled isothermally. They have used finite difference scheme to study variation of Nusselt number with Darcy-Rayleigh number, aspect ratio and radii ratio. Nithiarasu, Seetharamu, Sundarajan [7] have worked on natural convection in axisymmetric porous cavity using finite element method. Sankar, Kim, Lopez and Do [8] have studied thermosolutal natural convection in porous annulus and effect of Rayleigh number, Lewis number, buoyancy ratio and radius ratio. Grossana et al [9] have studied effect of internal heat generation and magnetic field on natural convection in porous cavity.

Nomenclature

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Asp	Aspect Ratio (H/D)
D	ro-ri, width of annulus
Da	Darcy number
g	Acceleration due to gravity (m/s ²)
Gr	Grashoff number
h	Location of centre of source (m)
H	Height of cylinder (m)

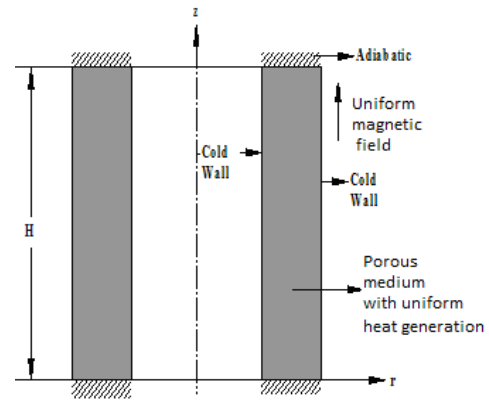
Ha	Hartmann number
k	Thermal conductivity (W/m-K)
K	Permeability of porous media (m ²)
l	Dimensional length of source (m)
L	Dimensionless location of centre of heat and solute source (h/H)
Nu	Nusselt number
Pr	Prandtl number
q ₀	Constant heat generation(W/m ³)
r,z	Cylindrical coordinates (m)
r,z	Non dimensional coordinates
Ra *	Rayleigh number
Ra	Darcy-Darcy-Rayleigh number
Radr	Radius Ratio
ro,ri	Outer radius, inner radius (m)
T,T	Dimensional and Non dimensional Temperature (K)
u,w	Velocity in r. z direction (m/s)

Greek symbols

α_T	Thermal diffusivity(m ² /s)
β_T	Thermal expansion coefficient (1/K)
ϵ	Non dimensional length of source (l/H)
ψ	Stream function
Ψ	Non dimensional Stream function
ρ	Density (Kg/m ³)
ν	Kinematic viscosity (m ² /s)

Subscripts

i	inner wall
o	outer wall



b) Uniform vertical magnetic field

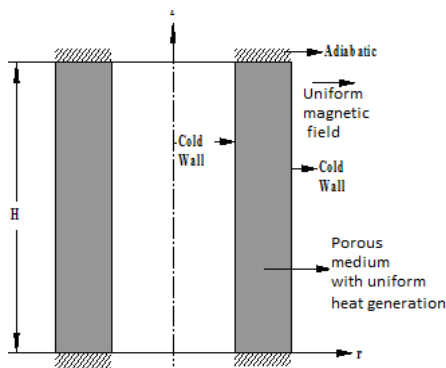
Fig. 1 Physical model and coordinate system

Ahmed, Badruddin, Kanesan, Zainal, Ahamed [10] have worked on mixed convection in porous annular cylinder with isothermal conditions on inner and outer wall considering thermal non equilibrium. Finite element method has been used to obtain results. Chamkha and Khaleed [11] have studied natural convective heat and mass transfer in a semi infinite plate under the influence of magnetic field subjected to constant flux boundary condition. Wang, Zeng , Huang , Wang , Ozoe . [12] have worked on natural convection in inclined porous enclosure subjected to high magnetic field for various field strength, inclination angle and Darcy number. Mahmud and Fraser[13] have studied heat transfer in square porous cavity with entropy generation under influence of magnetic field. Bejan and Nield [14] have consolidated literature in porous media in their book 'Convection in porous media'. 'Fundamentals of Finite Element Method for Heat and Fluid Flow' by Lewis, Nithiarasu and Seetharamu [15] provides the method of FEM applied to porous media. There are works on MHD in porous media. However most of them are restricted to two dimensional porous enclosure or cavities. There are very few works on MHD in porous annulus. To the best of author's knowledge, there is no work on effect of heat generation in vertical porous annulus subjected to uniform vertical and horizontal magnetic field for different Darcy Rayleigh number, aspect ratio, radius ratio, Hartmann number.

Present study deals with effect uniform heat generation, aspect ratio, radius ratio, Hartmann number (field strength), magnetic field direction on Nusselt number, isotherms and streamlines.

2. MATHEMATICAL MODELLING AND FORMULATION

The analysis performed is for a steady, laminar, incompressible natural convection for fluid saturated annulus porous solid matrix with uniform volumetric porosity. Fluid is assumed to be Newtonian with negligible viscous dissipation. The convecting fluid and the solid matrix have been considered to be in local thermal equilibrium. Boussinesq approximation for linear density variation with local fluid



a) Uniform horizontal magnetic field

temperature was assumed to be the cause of the buoyancy force in the convecting fluid.

Governing equation for free convection in annular cylinder is given as

Momentum equation

For uniform radial magnetic field

$$\frac{\partial w}{\partial r} - \frac{\partial u}{\partial z} = \frac{gK}{\nu} \left(\beta_T \frac{\partial T}{\partial r} \right) - \frac{\sigma_0 \beta_0 \omega^2 k}{u} \left[\frac{\partial u}{\partial r} \right] \quad (1)$$

For uniform axial magnetic field

$$\frac{\partial w}{\partial r} - \frac{\partial u}{\partial z} = \frac{gK}{\nu} \left(\beta_T \frac{\partial T}{\partial r} \right) + \frac{\sigma_0 \beta_0 \omega^2 k}{u} \left[\frac{\partial u}{\partial z} \right] \quad (2)$$

Where,

$$\rho = \rho_\infty (1 - \beta_T (T - T_\infty))$$

Energy equation

$$\rho C_p \left(u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) = \left[k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right) \right] + \frac{q_0}{\rho C_p} \quad (3)$$

Corresponding boundary conditions are

At $r=r_i$,
 $u=0, T=T_0$

At $r=r_o$
 $u=0, T=T_0$

Non dimensionalising various parameters

$$\bar{T} = \frac{(T - T_\infty)k}{T_*}, \quad R = \frac{r}{D}, \quad Z = \frac{z}{D}$$

$$\bar{\Psi} = \frac{\Psi}{\alpha D}, \quad Ra = \frac{gK(\beta T)qD^2}{\nu k(\alpha_T)}, \quad \alpha = \frac{k}{\rho C_p}$$

Substituting dimensionless parameters gives following non dimensional Partial differential equation

Momentum equation

For radial magnetic field

$$\frac{\partial^2 \bar{\Psi}}{\partial \bar{z}^2} + R \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \bar{\Psi}}{\partial R} \right) = R.Ra. \left(\frac{\partial \bar{T}}{\partial R} + N. \frac{\partial \bar{C}}{\partial R} \right) - Ha2.R \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \bar{\Psi}}{\partial R} \right) \quad (4)$$

For axial magnetic field

$$\frac{\partial^2 \bar{\Psi}}{\partial \bar{z}^2} + R \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \bar{\Psi}}{\partial R} \right) = R.Ra. \left(\frac{\partial \bar{T}}{\partial R} + N. \frac{\partial \bar{C}}{\partial R} \right) - Ha2. \frac{\partial^2 \bar{\Psi}}{\partial \bar{z}^2} \quad (5)$$

Energy equation

$$\frac{1}{R} \left[\frac{\partial \bar{\Psi}}{\partial R} \frac{\partial \bar{T}}{\partial Z} - \frac{\partial \bar{\Psi}}{\partial Z} \frac{\partial \bar{T}}{\partial R} \right] = \frac{1}{R} \frac{\partial}{\partial R} \left(R. \frac{\partial \bar{T}}{\partial R} \right) + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} + 1 \quad (6)$$

Boundary conditions are given as

At $R=R_i$.
 $\bar{\Psi}=0, \bar{T}=0$

At $R=R_0$
 $\bar{\Psi}=0, \bar{T}=0$

The average Nusselt and Sherwood number on surface of heat and solute source at inner wall is given as

$$\bar{Nu} = - \int_0^1 \frac{\partial \bar{T}}{\partial \bar{r}} \bar{dz} \quad (7)$$

3. SOLUTION METHODOLOGY AND GOVERNING EQUATIONS

The solution of stream function and energy equations gives the effect of various parameters on fluid flow and heat transfer in presence of magnetic field. In the present study, Finite Element Method (FEM) is used for all the case studies.

The computational domain bounded by the region of the porous wall is meshed. Triangular elements have been used. The non dimensional radius varies from 0.2 to 1.2. The non dimensional height varies from 0 to 5. A 31x31 matrix has been used. The region consists of 1800 elements and 961 nodes. MATLAB code has been developed for present model and has been validated as shown below.

3.1 Grid independence study

Grid independence study has been carried out for 3 grids- 21x21, 31x31, 41x41. Study has been carried out for same constant temperature on inner and outer wall of annulus with uniform internal heat generation. A uniform horizontal magnetic field has been applied. Hartmann number as 3 has been used. Aspect ratio and radius ratio have been taken as 2. Darcy-Rayleigh number as 500 has been used for study. Local Nusselt number for all three grids has been found to be same. Isotherms and streamlines have also been found to be similar. Hence 31x31 grid has been chosen because 41x41 grid takes higher computational time while it is finer than 21x21 and takes similar computational time.

3.2 Validation

In order to verify accuracy of current numerical method, results have been compared with that of standard results from available literature ([5],[14]). Average Nusselt number for different Darcy-Rayleigh number and radius ratios have been obtained for natural convection driven by thermal buoyancy alone for heater along entire inner wall for aspect ratio=1. Sankar et al (2011) [9] have used implicit finite difference scheme to solve the same problem and have written a

FORTRAN code to solve governing equations. For Present study, Finite element method has been used and MATLAB has been used to write codes to solve governing differential equations. Results have been compared with that of Sankar et

al [9] and Prasad (1986) [17]. From TABLE I, an overall good agreement can be observed between present study and that of Sankar et al [9] and Prasad [14].

TABLE I
 COMPARISON OF NUSSLET NUMBER WITH SANKAR [5] AND PRASAD[14]

Radius Ratio	Darcy-Darcy-Rayleigh number	Prasad (1986) [14]	Sankar, Park, Lopez,Do (2011)	Present study
2	1000	6.4934	6.4815	6.5112
	10000	16.0498	16.0271	16.1548
3	1000	7.1659	7.1804	7.0288
	10000	17.2691	17.2226	16.9866
5	1000	8.0036	8.0262	7.9004
	10000	18.8055	18.8631	18.286
10	1000	9.3975	9.4452	9.689
	10000	20.7498	20.8325	20.8488

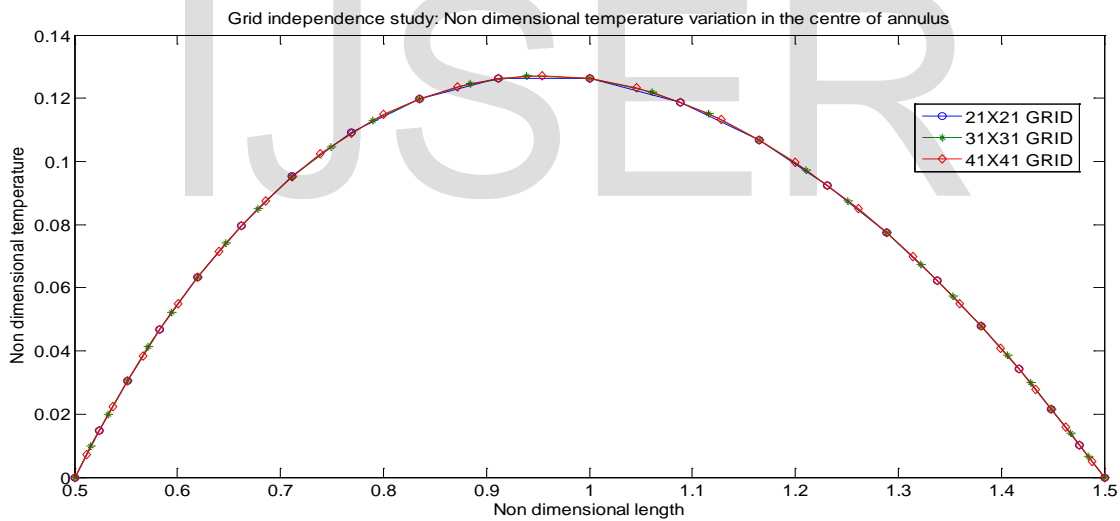
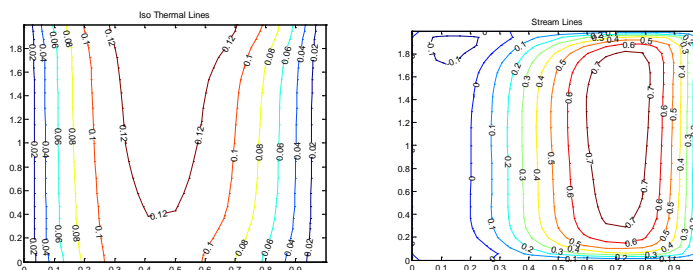


Fig. 2 Variation of non dimensional temperature in radial direction at centre of cavity for various grids



a) 21X21

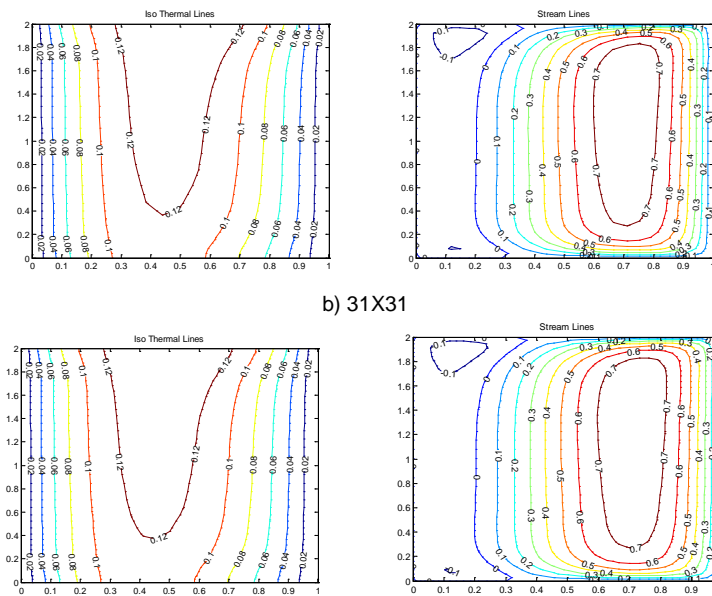
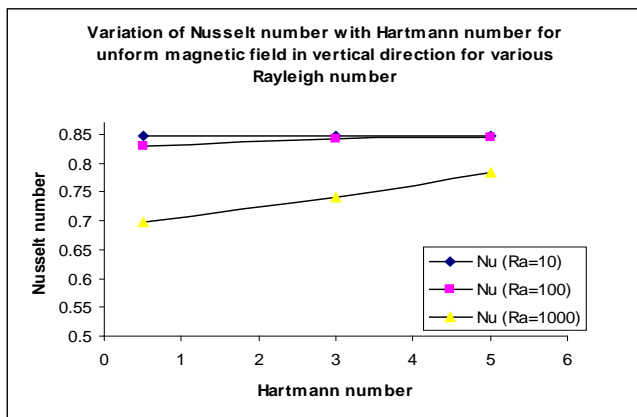


Fig. 3 Comparison of contours for different grids

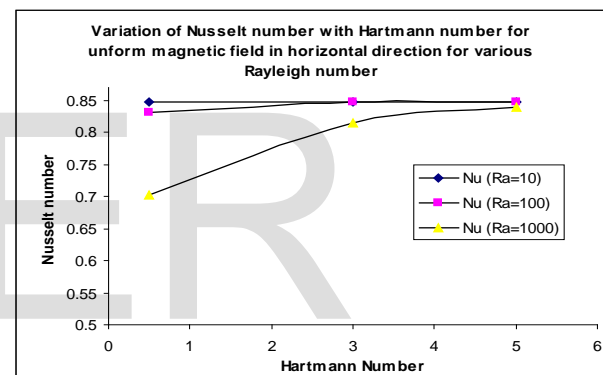
4. RESULTS AND DISCUSSION

In the following section, influence of strength and orientation of uniform magnetic field on heat transfer has been studied. Study has been carried out for constant temperature boundary condition. Normalized porosity has been kept at unity. Effect of uniform internal heat generation has been studied. Darcy-Rayleigh number has been varied from 10 to 1000. Hartmann number has been varied from 0 to 5 for both uniform horizontal and vertical field. For various Hartmann number for each orientation, aspect ratio and radius ratio has been varied. Aspect ratio and Radius ratio have also been varied, keeping other parameters constant, as $1 < Asp < 7$ and $1 < Radr < 7$. Flow fields and temperature gradients have been illustrated using streamlines and isotherms. Nusselt number has been obtained for all the cases.

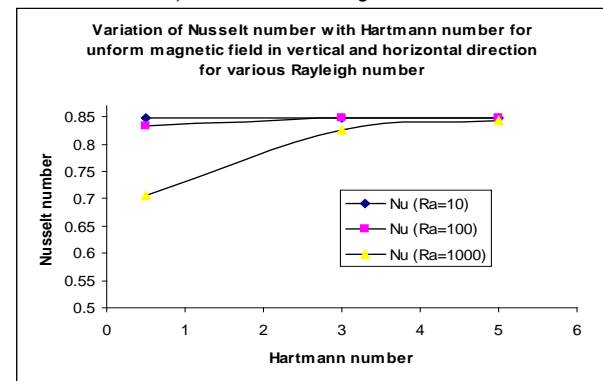
4.1 Effect of Hartmann Number



a) Uniform axial magnetic field



b) Uniform radial magnetic field



c) Uniform radial and axial magnetic field

Fig. 4 Variation of Nusselt number with Hartmann number for various magnetic field orientation

Hartmann number represents relative strength of Lorentz force to viscous force. It represents relative strength of magnetic field force. In present study, Hartmann number has been varied from 0.5 to 5. Uniform magnetic field has been applied in horizontal and vertical direction and its effect has been observed on Nusselt number, streamlines and isotherms.

From Nusselt number plots, it can be inferred that Nusselt number increases with increase in Hartmann number. This can also be seen from isotherms and streamlines. For lower Hartmann number in both axial and radial direction, buoyancy driven flow is dominant with low circulation near wall. Temperature gradients are also low near hot wall for low Hartmann number. As Hartmann number is increased, for both axial and radial direction, circulation increases near walls. Also thermal gradients become high near wall. A general observation which can be made is that circulation in core of annulus is reduced in presence of magnetic field. However fluid flow increases near wall. This is because due to strong magnetic field, hot fluid is attracted near wall which leads to increased thermal gradients. This means that walls are hotter than other regions. As Nusselt number depends on thermal gradients near wall, it increases with increase in intensity of magnetic field. Another comparison can be made between effect of Hartmann number in axial and radial direction. For low Hartmann number, temperature gradients are similar for both axial and radial magnetic field, with uniform gradient along wall and little gradient in the core of annulus. For high value of Hartmann number, stronger gradients near wall can be observed for axial field as it tends to attract hot fluid near wall. For field in vertical direction, thermal gradients increase but not as much as horizontal field. Nusselt number values are higher for uniform magnetic field in horizontal direction compared to field in vertical direction

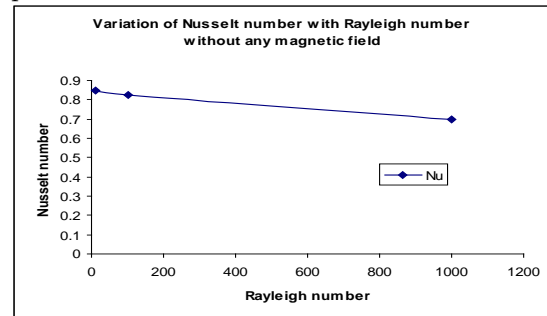
4.2 Effect of Rayleigh number

While Hartmann number represents ratio of Lorentz force to viscous force, Rayleigh number represents ratio of buoyant force to viscous force. Lorentz force counters buoyant force with increase in Hartmann number. Instead of buoyancy driven flow which rises up and recirculates, flow tends to get attracted to walls due to magnetic field. With increase in Rayleigh number, intensity of heat generation increases which provides thermal energy to fluid to overcome magnetic force.

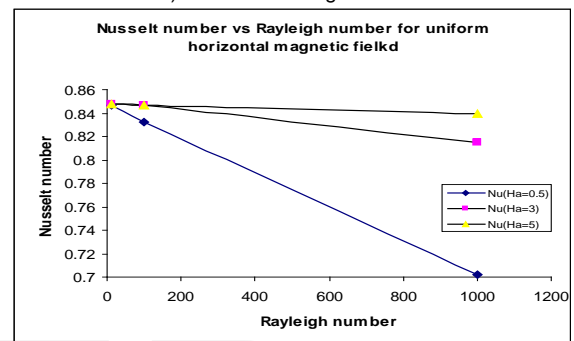
In general, increase in Rayleigh number leads to reduction in Nusselt number value for a constant value of Hartmann number. This can be seen from plots comparing Nusselt number with Rayleigh number for various Hartmann number. While Nusselt number is same for low Rayleigh number for all Hartmann number, for high Rayleigh number there is a significant change. Nusselt number is higher for axial magnetic field than radial field.

Differences can also be spotted in streamlines and isotherms with increase in Rayleigh number. For low Hartmann number, as Rayleigh number increases, streamlines move away from wall and intensity of circulation in core increases. For large Hartmann number, there is little difference in streamlines due to high Lorentz force. However core circulation increases with increase in Rayleigh number. Isotherms show significant difference for both low and high Rayleigh number. With increase in Rayleigh number, intensity of uniform heat

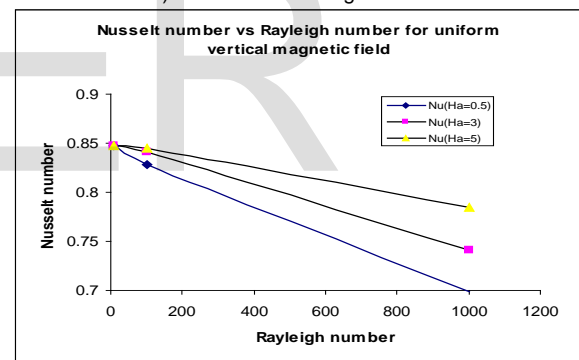
generation increases which lead high temperature in the centre of annulus and low temperature along walls. This can be seen from dip in isotherms in core of annulus.



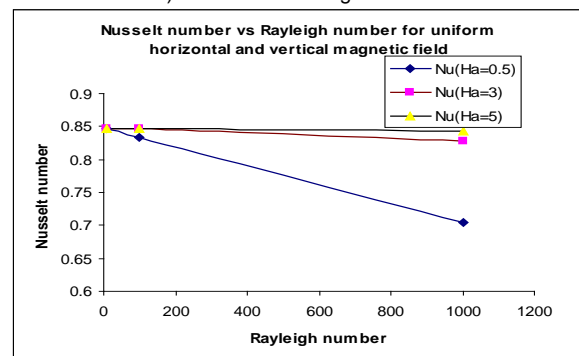
a) Absence of magnetic field



b) Uniform radial magnetic field

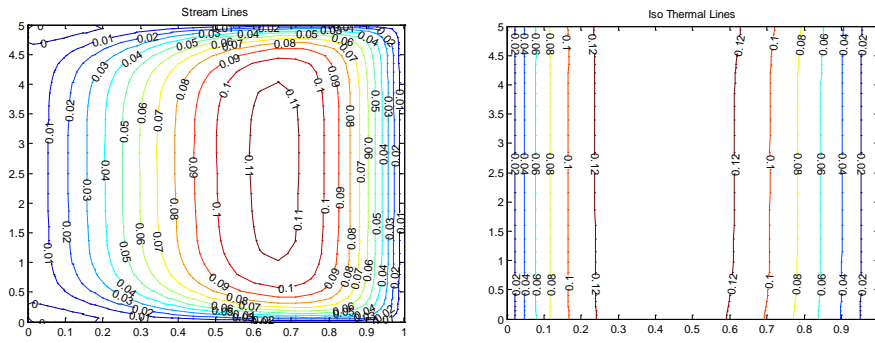


c) Uniform axial magnetic field

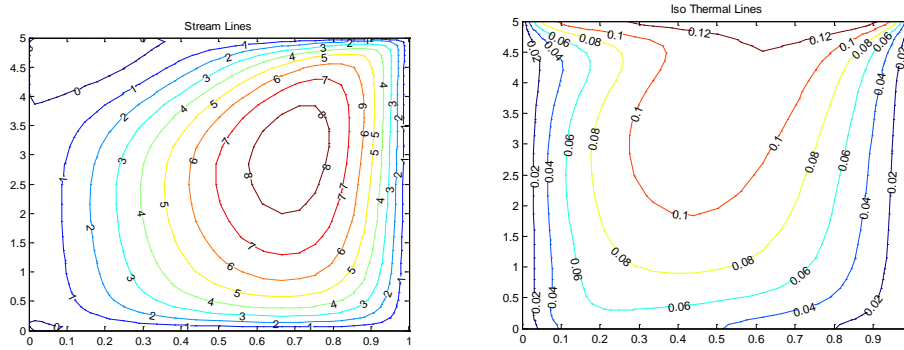


d) Combined axial and radial magnetic field

Fig. 5 Variation of Nusselt number with Rayleigh number

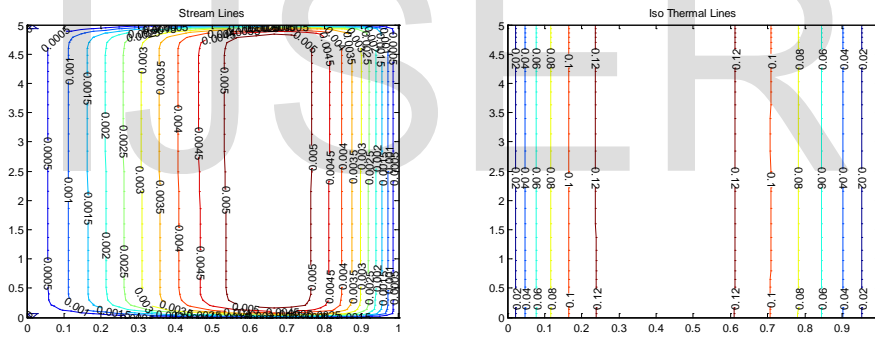


a) $Hax=0.5, Ra=10$

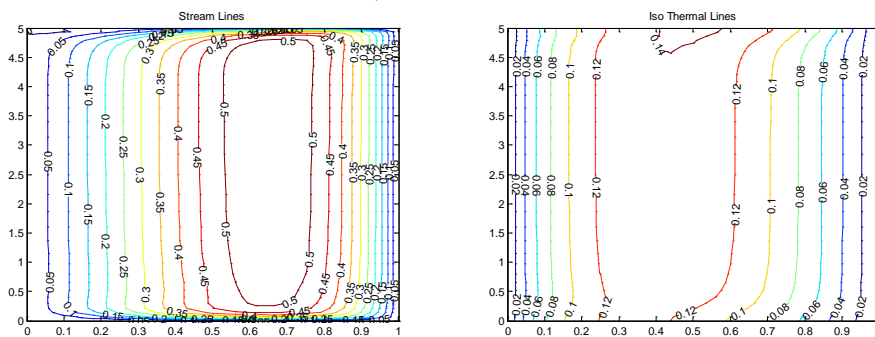


b) $Hax=0.5, Ra=1000$

Fig. 6 Variation of Streamlines and isotherms for $Hax=0.5$ with Rayleigh number

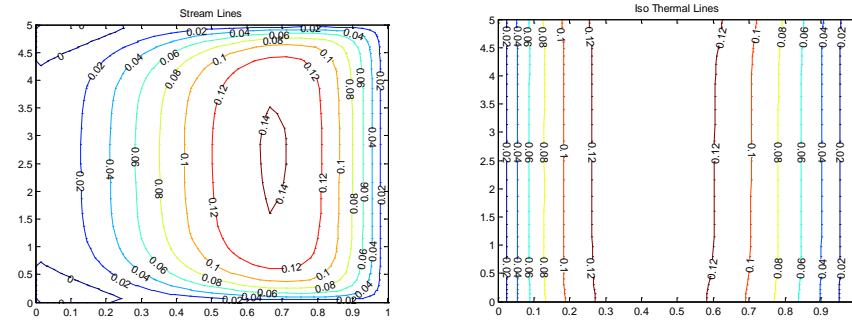


a) $Hax=5, Ra=10$

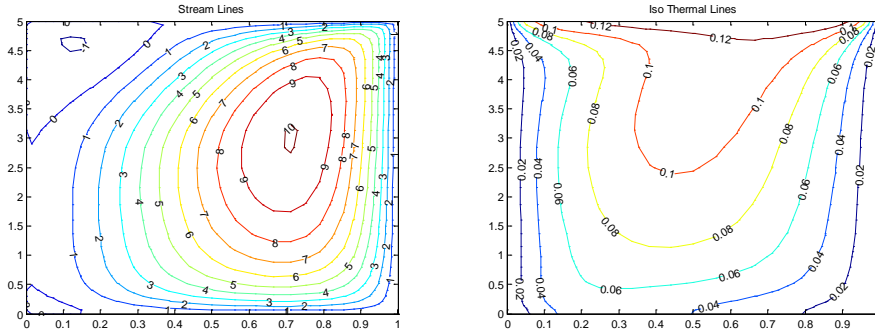


b) $Hax=5, Ra=1000$

Fig. 7 Variation of Streamlines and isotherms for $Hax=5$ with Rayleigh number

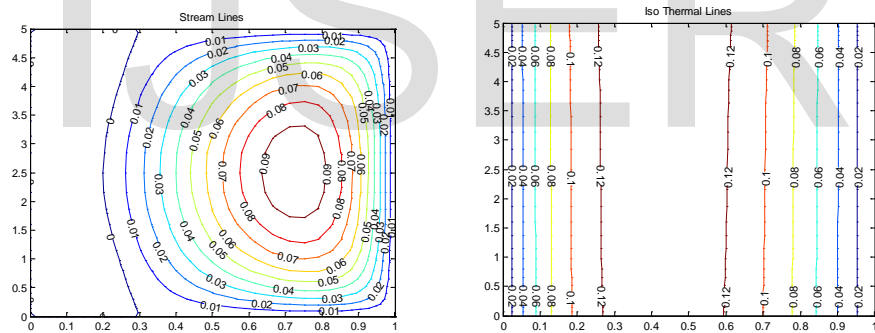


a) Hay=0.5, Ra=10

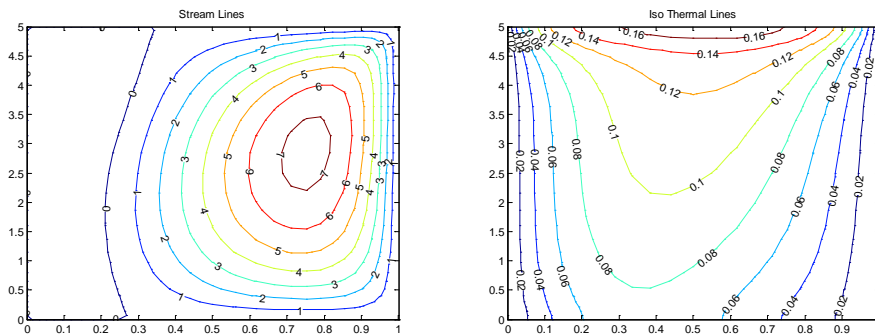


a) Hay=0.5, Ra=1000

Fig. 8 Variation of Streamlines and isotherms for Hay=0.5 with Rayleigh number



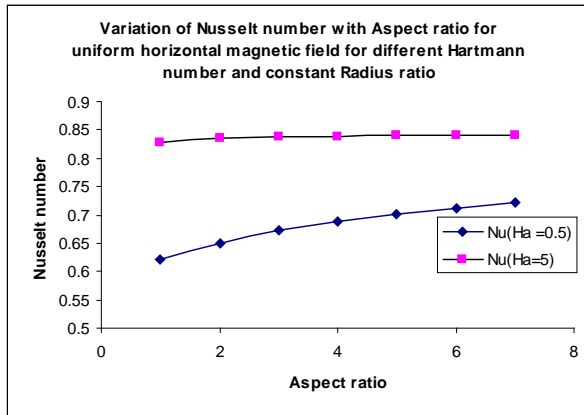
a) Hay=5, Ra=10



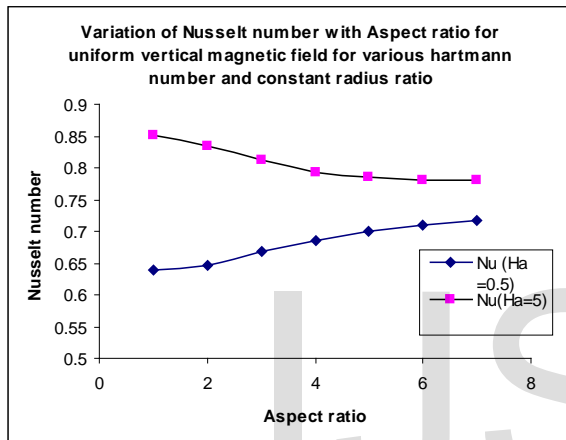
a) Hay=5, Ra=1000

Fig. 9 Variation of Streamlines and isotherms for Hay=5 with Rayleigh number

4.3 Effect of aspect ratio and radius ratio

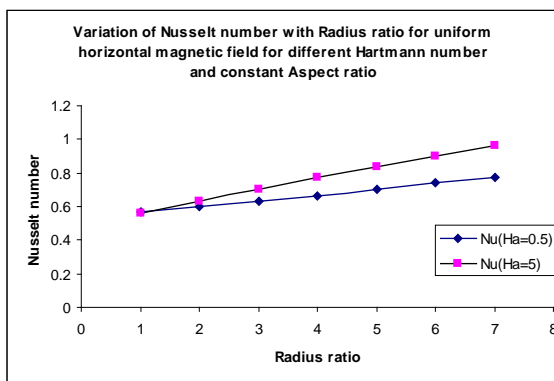


a) Radial magnetic field

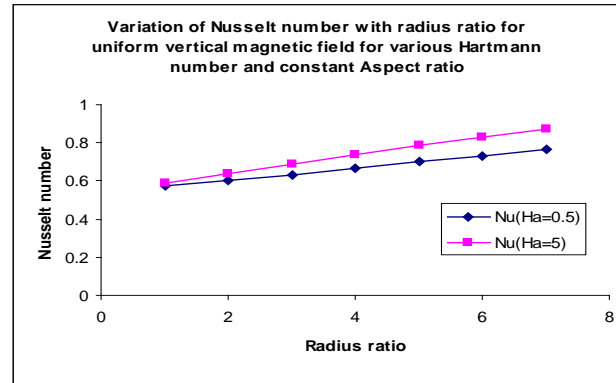


b) Radial magnetic field

Fig. 10 Variation of Nusselt number with aspect ratio



a) Radial magnetic field

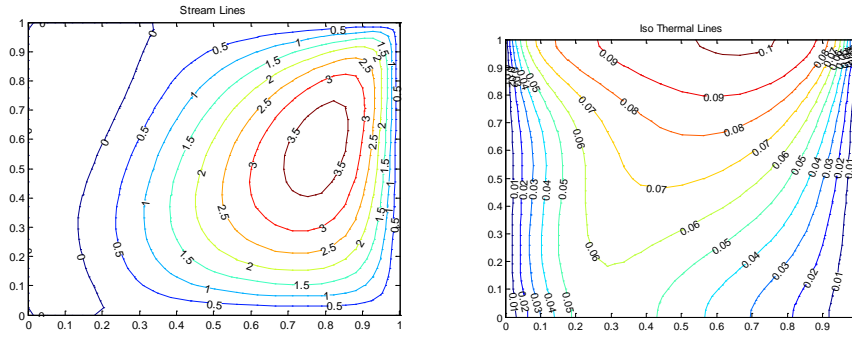


b) Axial magnetic field

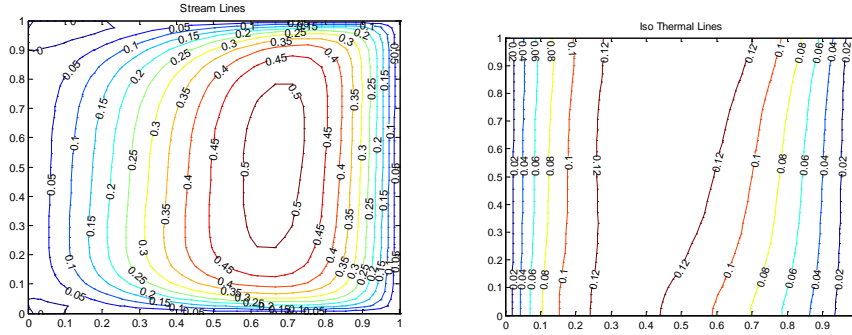
Fig. 11 Variation of Nusselt number with radius ratio

Nusselt number has been found to increase with increase in radius ratio for all values of Hartmann number. Increasing radius ratio leads to increase in width of annulus for same height. For shorter height, thermal gradients are strong and with increase in width, a larger volume of fluid becomes associated with strong gradients leading to high Nusselt number. For low radius ratio a bicellular flow can be observed, one associated with each wall. Intensity of circulation is weak for left wall. This flow becomes unicellular with increase in radius ratio and intensity of circulation increases spanning the entire annulus. Thermal gradients are almost symmetrical for low radius ratio. However they shift towards inner wall for high radius ratio and there is an increase in thermal gradients.

Variation with aspect ratio presents an interesting phenomenon. For low Hartmann number, Nusselt number increases with increase in aspect ratio for both uniform vertical and horizontal magnetic field. For higher Hartmann number, Nusselt number has been observed to be almost constant for horizontal magnetic field while it reduces with increase in aspect ratio for vertical magnetic field. For uniform horizontal magnetic field, flow circulation is shifted towards inner wall for low aspect ratio. As aspect ratio is increased, flow spans entire volume for both low and high Hartmann number which explains increase in Nusselt number. For vertical magnetic field, for low aspect ratio, bicellular counter rotating flow has been observed. As aspect ratio is increased, flow becomes unicellular with little circulation in inner wall. For high Hartmann number, Nusselt number decreases with increase in aspect ratio. For horizontal field, due to magnetic force along wall, fluid gets attracted. So the change with aspect ratio for high Hartmann number is negligible. However the effect is more pronounced for vertical field where there is no force attracting fluid towards wall leading to lower gradients.

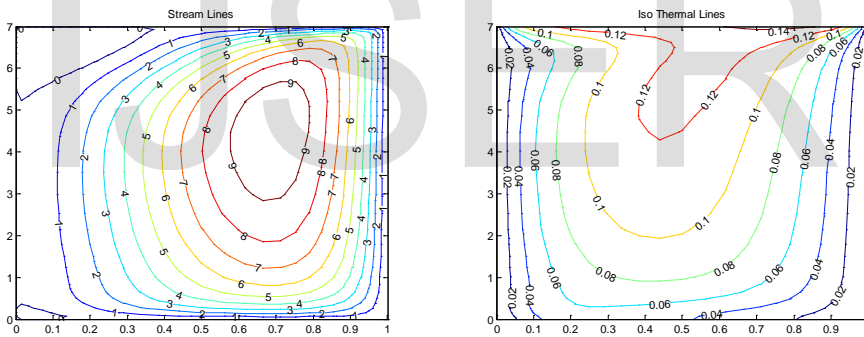


a) $Asp=1$, $Hax=0.5$

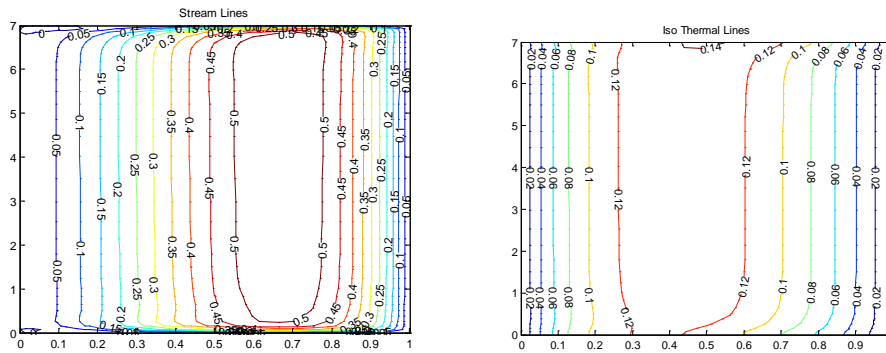


b) $Asp=1$, $Hax=5$

Fig. 12 Variation of Streamlines and isotherms for $asp=1$ with Hartmann number

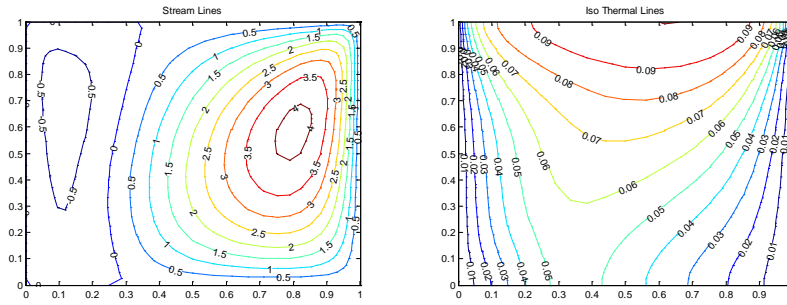


a) $Asp=7$, $Hax=0.5$

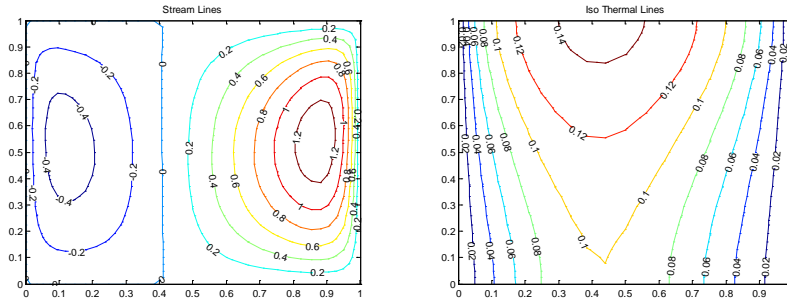


b) $Asp=7$, $Hax=5$

Fig. 13 Variation of Streamlines and isotherms for $asp=7$ with Hartmann number

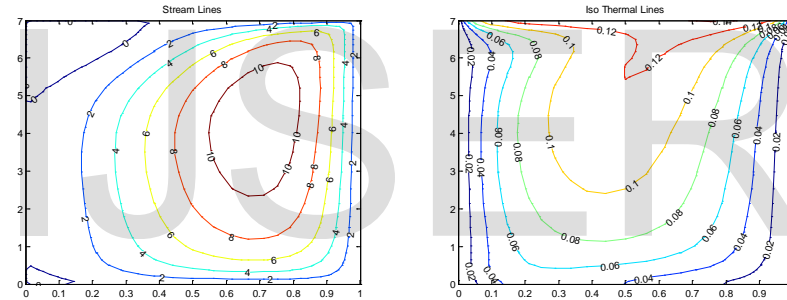


a) Asp=1, Hay=0.5

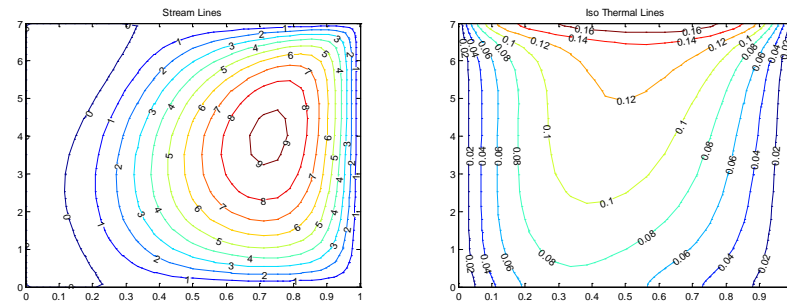


b) Asp=1, Hay=5

Fig. 14 Variation of Streamlines and isotherms for asp=1 with Hartmann number

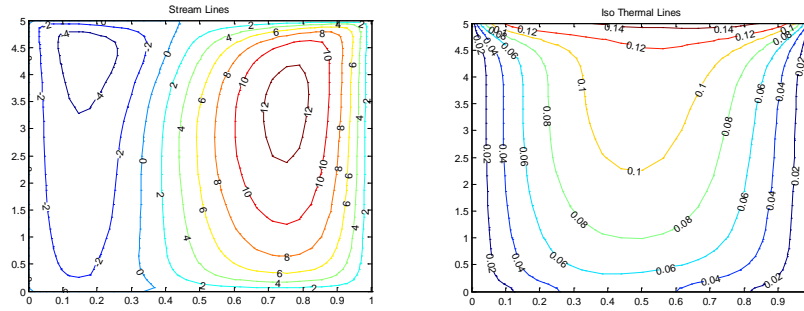


a) Asp=7, Hay=0.5

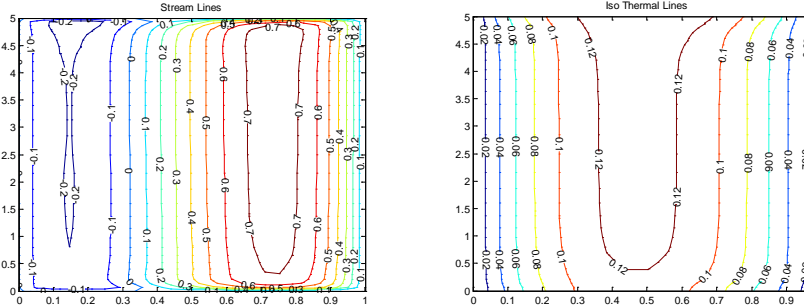


b) Asp=7, Hay=5

Fig. 15 Variation of Streamlines and isotherms for asp=7 with Hartmann number

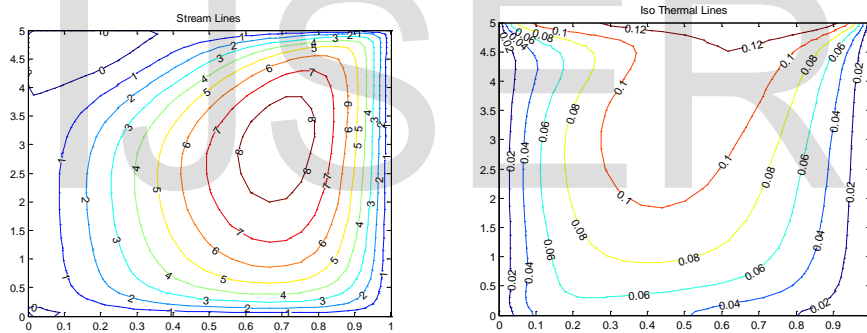


a) Radr=1, Hax=0.5

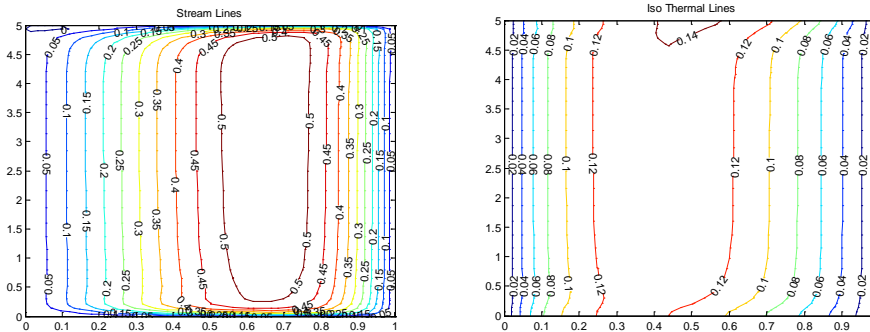


b) Radr=1, Hax=5

Fig. 16 Variation of Streamlines and isotherms for radr=1 with Hartmann number

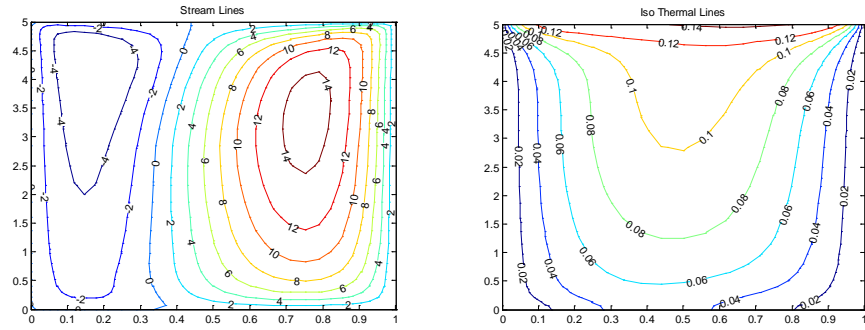


a) Radr=7, Hax=0.5

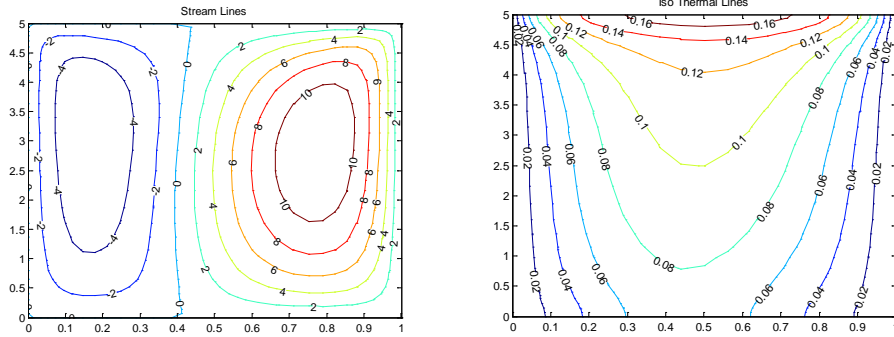


b) Radr=7, Hax=5

Fig. 17 Variation of Streamlines and isotherms for radr=1 with Hartmann number

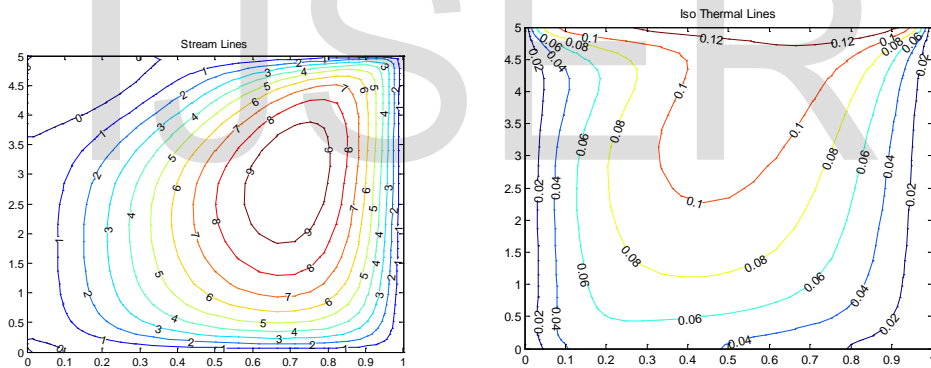


a) Radr=1, Hay=0.5

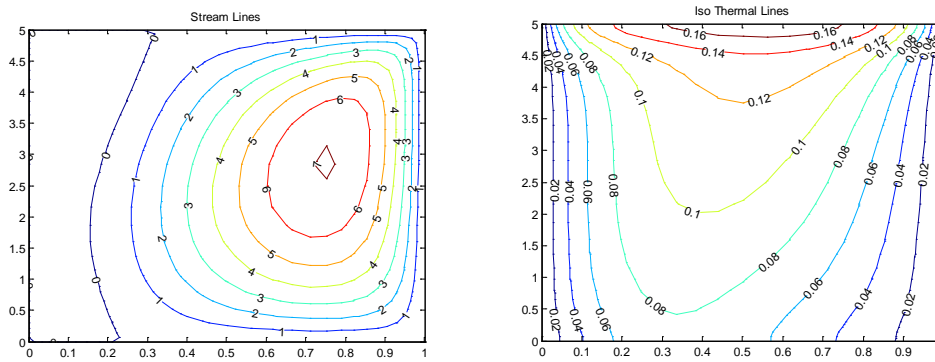


b) Radr=1, Hay=5

Fig. 17 Variation of Streamlines and isotherms for radr=1 with Hartmann number



a) Radr=7, Hay=0.5



a) Radr=7, Hay=5

Fig. 18 Variation of Streamlines and isotherms for Radr=7 with Hartmann number

5 CONCLUSION

Present study deals with natural convection in porous annulus with internal heat generation in presence of uniform horizontal and vertical magnetic field. With increase in Rayleigh number or magnetic field strength, Nusselt number has been found to increase due to increased gradient along wall. Increasing Rayleigh number or intensity of generation diminishes these thermal gradients leading to reduction in Nusselt number. For horizontal magnetic field, Nusselt number has been found to increase with aspect ratio for both low and high field strength. For vertical magnetic field, for low field strength, Nusselt number increases with aspect ratio while it decreases for high field strength. For all cases, Nusselt number has been found to increase with radius ratio.

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